Modal Logic

Final Exam | Phil 76600 | Spring 2023

Do any four of the following.

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1. In the propositional modal logic K, two of the following are provable, one is not. Give tableau proofs for those that are, and a counter-model for the one that is not.

   (a) $\Box(P \supset \Box\Box R) \supset \Box(P \supset \Box(\Diamond Q \supset \Diamond R))$

   (b) $\Box(P \supset \Box\Box R) \supset \Box(P \supset \Box(Q \supset \Diamond R))$

   (c) $\Box\Box(Q \supset \Diamond R) \supset \Box(P \supset \Box(Q \supset \neg\Box\neg R))$

2. You are to evaluate the validity of the following formula, where the underlying propositional modal logic is K.

   $$(\forall x)\Box[P(x) \wedge Q(x)] \supset [\Diamond(\forall x)P(x) \vee \Box(\exists x)Q(x)]$$

   Decide whether the formula is valid for actualist quantification (varying domains), and also for possibilist quantification (constant domains). If valid, give a tableau proof using the appropriate rules. If not valid, give a counter-example using the appropriate condition on domains.

3. Consider the following two formulas. To make reading a little easier, we have written Px instead of $P(x)$.

   (a) $\langle\lambda y.\Box\langle\lambda x.x = y\rangle(c)\rangle(c) \supset [\Diamond\langle\lambda x.Px\rangle(c) \supset \langle\lambda x.\Diamond Px\rangle(c)]$

   (b) $\langle\lambda y.\Box\langle\lambda x.x = y\rangle(c)\rangle(c) \supset [\langle\lambda x.\Diamond Px\rangle(c) \supset \Diamond\langle\lambda x.Px\rangle(c)]$

   Using K tableau rules *where c might not designate at every possible world*, which are provable? Give the proofs for those cases in which a proof exists. (Don't prove the ones that are not provable.)

4. This one involves classical logic only. Give a tableau proof of the following, using Russell's way of handling definite descriptions.

   $$\langle\lambda x.Q(x)\rangle(\imath x.P(x)) \supset (\forall x)[P(x) \supset Q(x)]$$

5. Consider the following formulas, where the modal logic is K.

   (a) $\langle\lambda x.P(x)\rangle(\imath x.\Box P(x)) \supset \langle\lambda x.\Box P(x)\rangle(\imath x.\Box P(x))$

   (b) $\langle\lambda x.P(x)\rangle(\imath x.\Box P(x)) \supset \Box\langle\lambda x.P(x)\rangle(\imath x.\Box P(x))$

   Assume *constant domains*. Find one that is provable and give a proof. Use Russell's way of handling definite descriptions. Tell me what you think of the other one. (Be polite).