Corrections to
Set Theory and the Continuum Problem (revised edition)

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These are corrections to the edition published by Dover in 2010.

Page 23 (Error found by David Feuer) Exercise 5.5(d) in Chapter 2 asks the reader to show $B - (A - B) = \emptyset$. It should be to show $B - (A - B) = B$.

Page 23 (Error found by David Feuer) Text in §6 reads A_6 [**Power aet axiom**]. It should read A_6 [**Power set axiom**].

Page 25 (Error found by Gavin Rebeiro) On pg. 25 line 36 (5th line of last paragraph), "if $x \neq y$ then $F(x) \neq F(Y)$" should be "... $F(x) \neq F(y)$".

Page 39 (Error found by Gavin Rebeiro) On pg. 36, last paragraph on page. "Let L be the class of all elements x such that x is a proper subclass of all elements of A" should be "Let L be the class of all elements x *in* M such that x is a proper subclass of all elements of A.

Page 44 (Error found by Martin Epstein) In Exercise 8.4 (b) $M(x, y^+) = M(x, y) + y$ should be replaced with $M(x, y^+) = M(x, y) + x$. Also $x \cdot y^+ = (x \cdot y) + y$ should be replaced with $x \cdot y^+ = (x \cdot y) + x$.

Page 51 (Error found by Grigory Olkhovikov) Exercise 1.2 is incorrect as asked. Take A to be the set of negative integers under the natural linear order $\leq$. Every proper lower section of A has a strict upper bound, but its ordering is not a well ordering. The exercise can be corrected by including the condition that A has a least element

Page 55 (Error found by Hal Prince) line 6: "every y in M" should be "every y in C".

Page 60 (Error found by Allen David Boozer) Exercise 4.3 should be deleted. There is a reference to this Exercise in the Remark at the top of page 63, and this reference should also be deleted.

Page 66 (Error found by Stuart Newberger) Definition 7.1 is incorrect as stated. It should read as follows.

For any sets y and x, we will say that y is *closed* (under g) *relative to* x provided, for any $z \in y$, if $g(z) \in \mathcal{P}(x)$ then $g(z) \in y$. (Thus $(z \in y \wedge g(z) \subseteq x) \supset g(z) \in y$.)

Page 225-226 (Error found by Chang Soon Choi.) In the Remarks, it is not the case that $(f \approx_\lambda g) \supset [\![f \approx_\lambda g]\!]$ is $\mathsf{S4}$ valid generally, but it is valid in the particular $\mathsf{S4}$ models being constructed. Here is the argument. Suppose as an induction hypothesis that it is known for ordinals less than λ. It follows from the definition of $\approx_\lambda$ that if $\alpha < \lambda$ then $(f \approx_\alpha g) \supset (f \approx_\lambda g)$ is valid in these models. It follows from this, using general $\mathsf{S4}$ reasoning, that $\Box\Diamond(f \approx_\alpha g) \supset \Box\Diamond(f \approx_\lambda g)$ is also valid in these models, that is, $[\![f \approx_\alpha g]\!] \supset [\![f \approx_\lambda g]\!]$. Now if $p \Vdash (f \approx_\lambda g)$, then $p \Vdash (f \approx_\alpha g)$ for some $\alpha < \lambda$ (by definition). This implies $p \Vdash [\![f \approx_\alpha g]\!]$, and hence $p \Vdash [\![f \approx_\lambda g]\!]$.

Page 227 (Error found by Chang Soon Choi.) In the proof of Proposition 1.5, the limit ordinal case is incorrect. It uses an inference from $p \Vdash [\![f \approx_\lambda g]\!]$, where λ is a limit ordinal, to $p \Vdash [\![f \approx_\alpha g]\!]$, for some $\alpha < \lambda$, and this is not justified. Replace the limit ordinal case by the following.

Assume λ is a limit ordinal and every $\alpha < \lambda$ is good. Now suppose $p \Vdash [\![f \approx_\lambda g]\!]$ and $\lambda < \beta$; we must show $p \Vdash [\![f \approx_\beta g]\!]$.

We first show $(f \approx_\lambda g) \supset [\![f \approx_\beta g]\!]$ is valid in the model that has been constructed (note the absence of double square brackets in the antecedent). Well, suppose $q \Vdash (f \approx_\lambda g)$. Then $q \Vdash (f \approx_\alpha g)$ for some $\alpha < \lambda$, by definition of $\approx_\lambda$. It follows by the Remarks at the bottom of page 225 and the top of 226 that $q \Vdash [\![f \approx_\alpha g]\!]$. Since α is good, $q \Vdash [\![f \approx_\beta g]\!]$. Since q was arbitrary, we have shown the validity of $(f \approx_\lambda g) \supset [\![f \approx_\beta g]\!]$ in the model.

It now follows, by standard modal manipulations, that $\Box\Diamond(f \approx_\lambda g) \supset \Box\Diamond[\![f \approx_\beta g]\!]$ is also valid in the model, and hence we have the validity of $[\![f \approx_\lambda g]\!] \supset [\![f \approx_\beta g]\!]$, making use of Proposition 4.3, part 2. Since $p \Vdash [\![f \approx_\lambda g]\!]$, then $p \Vdash [\![f \approx_\beta g]\!]$.

Page 228 (Found by Grigori Mints) In the Remark just before Definition 1.8 it is asserted that $(f \in g) \equiv [\![f \in g]\!]$. The equivalence is not correct, but $(f \in g) \supset [\![f \in g]\!]$ is.

Page 229 (Problem found by Chang Soon Choi.) Lemma 2.1 says that if $p \Vdash [\![f \approx_\alpha g]\!]$ and $p \Vdash [\![g \approx_\alpha h]\!]$ then $p \Vdash [\![f \approx_\alpha h]\!]$. The proof is by induction on α. It begins by saying the cases where α is 0 or a limit ordinal are simple. In fact 0 is simple, but the limit ordinal case is not. Here is a proof for the limit ordinal case.

Let λ be a limit ordinal and assume the result holds for smaller ordinals. We begin by showing that if $p \Vdash (f \approx_\lambda g)$ and $p \Vdash [\![g \approx_\lambda h]\!]$ then $p \Vdash [\![f \approx_\lambda h]\!]$ (note the difference in the first item). So, suppose $p \Vdash (f \approx_\lambda g)$ and $p \Vdash [\![g \approx_\lambda h]\!]$. To show $p \Vdash [\![f \approx_\lambda h]\!]$ we show $p \Vdash \Box\Diamond(f \approx_\lambda h)$. Let q be any member of $\mathcal{G}$ such that $p\mathcal{R}q$; we must show there is some r with $q\mathcal{R}r$ so that $r \Vdash (f \approx_\lambda h)$.

Since $p \Vdash \Box\Diamond(g \approx_\lambda h)$ then $q \Vdash \Diamond(g \approx_\lambda h)$ and hence there is some r with $q\mathcal{R}r$ so that $r \Vdash (g \approx_\lambda h)$. By definition, $r \Vdash (g \approx_\alpha h)$ for some $\alpha < \lambda$. Without loss of generality we can assume α is a successor ordinal. Then $r \Vdash [\![g \approx_\alpha h]\!]$ by the remarks on pages 225-226.

Since $p \Vdash (f \approx_\lambda g)$ then $p \Vdash (f \approx_\beta g)$ for some $\beta < \lambda$ and again without loss of generality we can assume β is a successor ordinal. Then $p \Vdash [\![f \approx_\beta g]\!]$ by the remarks on pages 225-226 again. Since this formula begins with $\Box$, $r \Vdash [\![f \approx_\beta g]\!]$. Let γ be the larger of α and β. By Proposition 1.5, $r \Vdash [\![f \approx_\gamma g]\!]$ and $r \Vdash [\![g \approx_\gamma h]\!]$. Since $\gamma < \lambda$, by the induction hypothesis for the overall Lemma, $r \Vdash [\![f \approx_\gamma h]\!]$. Since γ is a successor ordinal, by the remarks on pages 225-226 again, $r \Vdash (f \approx_\gamma h)$, which is what we wanted.

Since p was arbitrary, we have shown the validity in our model of

$$(f \approx_\lambda g) \supset \left([\![g \approx_\lambda h]\!] \supset [\![f \approx_\lambda h]\!]\right).$$

Then by standard $\mathsf{S4}$ manipulations, this gives us the validity in our model of

$$\Box\Diamond(f \approx_\lambda g) \supset \Box\Diamond\left([\![g \approx_\lambda h]\!] \supset [\![f \approx_\lambda h]\!]\right).$$

By Proposition 4.4 of Chapter 16 we then have

$$\Box\Diamond(f \approx_\lambda g) \supset [\![g \approx_\lambda h \supset f \approx_\lambda h]\!]$$

and hence

$$[\![f \approx_\lambda g]\!] \supset \left([\![g \approx_\lambda h]\!] \supset [\![f \approx_\lambda h]\!]\right)$$

by using Proposition 4.5 of Chapter 16.

Page 234 (Problem found by Chang Soon Choi.) In line 8 of the proof of Proposition 3.3, "equivalently, $[\![\hat{s} \in \hat{t}]\!]$" should be changed to "and so $[\![\hat{s} \in \hat{t}]\!]$". Also in line 11 of the same proof, "But then $p \Vdash (a \,\varepsilon\, \hat{t})$, so a is $\hat{x}\ldots$" should be changed to "So a is $\hat{x}\ldots$".

Pages 239–240 (Problem found by Chang Soon Choi.) In the proof of Lemma 5.2 it is said that "(Recall that $(x \approx_\alpha y)$ and $[\![x \approx_\alpha y]\!]$ are equivalent.)" This is not the case. One should modify the condition that we need to express by a first-order formula so that the last part reads $\Box\Diamond(x \approx_\alpha y)$. Then, in the formula following "Now let $F(\mathcal{A}, p, f, g)$ be the formula:" the final clause should be changed from "$\langle s', x, y\rangle \in \mathcal{A}$" to "$\langle r', x, y\rangle \in \mathcal{A}$".

Page 241 (Problem found by Chang Soon Choi.) In the proof of Theorem 5.4, the atomic case should be modified. We know that $p \Vdash [\![f \in g]\!]$ iff $p \Vdash [\![(\exists w)(w \approx f \wedge w \,\varepsilon\, g)]\!]$ iff $p \Vdash \Box\Diamond(\exists w)([\![w \approx f]\!] \wedge [\![w \,\varepsilon\, g]\!])$, so in the atomic case, $F_\varphi(z, x, y)$ should be

$$(\forall z' \overleftarrow{\mathcal{R}} z)(\exists z'' \overleftarrow{\mathcal{R}} z')(\exists w \in \mathcal{D})(\mathsf{Equals}(z'', w, x) \wedge (\forall z''' \overleftarrow{\mathcal{R}} z'')(\exists z'''' \overleftarrow{\mathcal{R}} z''')\langle z'''', w\rangle \in y)$$

Also in the final part of the proof take $F_\varphi(z, x_1, \ldots, x_n)$ to be the following: $(\forall z' \overleftarrow{R} z)(\exists z'' \overleftarrow{R} z')\neg F_\psi(z'', x_1, \ldots, x_n)$.

Page 264 (Problem found by Jason Parker) The remarks at the end of the first paragraph are incorrect. First, a few useful observations: using Definition 3.1 on page 233, one has the following.

$$\begin{aligned}
\hat{0} &= \emptyset \\
\hat{1} &= \mathcal{G} \times \{\hat{0}\} \\
\hat{2} &= \mathcal{G} \times \{\hat{0}, \hat{1}\} \\
&= \hat{1} \cup (\mathcal{G} \times \{\hat{1}\}) \\
\hat{3} &= \mathcal{G} \times \{\hat{0}, \hat{1}, \hat{2}\} \\
&= \hat{2} \cup (\mathcal{G} \times \{\hat{2}\}) \\
&\vdots \\
\hat{\omega} &= \mathcal{G} \times \{\hat{0}, \hat{1}, \hat{2}, \ldots\}
\end{aligned}$$

Now, here is Parker's argument.

"It is claimed that we can show that if $p \Vdash [\![a \subseteq \hat{\omega}]\!]$, then for some $b \subseteq \mathcal{G} \times \hat{\omega}$, $p \Vdash [\![a \approx b]\!]$. But this would entail that $b \in \mathcal{D}^{\mathcal{G}}$, which does not seem possible. For suppose $b \subseteq \mathcal{G} \times \hat{\omega}$. If b were a member of $\mathcal{D}^{\mathcal{G}}$, then $b \in R^{\mathcal{G}}_{\alpha+1}$ for some least ordinal α. Then $b \subseteq \mathcal{G} \times R^{\mathcal{G}}_{\alpha}$. Now since $b \subseteq \mathcal{G} \times \hat{\omega}$, it follows that any $x \in b$ is of the form $\langle p, \langle q, \hat{n}\rangle\rangle$ for some $p, q \in \mathcal{G}$ and $n \in \omega$. So $\langle q, \hat{n}\rangle \in R^{\mathcal{G}}_{\alpha}$. So there is some least ordinal β such that $\langle q, \hat{n}\rangle \in R^{\mathcal{G}}_{\beta+1}$, whereby

$\langle q, \hat{n}\rangle \subseteq \mathcal{G} \times R_\beta^{\mathcal{G}}$, which is clearly false. So it seems that it cannot be that $b \in \mathcal{D}^{\mathcal{G}}$ if $b \subseteq \mathcal{G} \times \hat{\omega}$."

The problem sentences at the end of paragraph 1, page 264, should be replaced with the following. "Now, this result can be improved, to establish that if $[\![a \subseteq \hat{\omega}]\!]$ is true at p then $[\![a \approx b]\!]$ is true at p for some $b \subseteq \mathcal{G} \times \{\hat{n} \mid n \in \omega\}$ (equivalently, for some $b \subseteq \hat{\omega}$). Consequently, to investigate the size of the power set of $\hat{\omega}$ in the modal model, we begin by investigating the actual power set of $\hat{\omega}$."

Here is the argument for the revised assertion above. Throughout, assume that $p \Vdash [\![a \subseteq \hat{\omega}]\!]$, meaning $p \Vdash [\![(\forall x)(x \in a \supset x \in \hat{\omega})]\!]$.

1. If $p\mathcal{R}p'$ and $p' \Vdash [\![x \in a]\!]$, then for some p'' with $p'\mathcal{R}p''$, $p'' \Vdash [\![x \approx \hat{n} \wedge \hat{n} \in a]\!]$ for some $n \in \omega$.

   Proof: Suppose $p\mathcal{R}p'$, and $p' \Vdash [\![x \in a]\!]$. Then $p' \Vdash [\![x \in \hat{\omega}]\!]$, and so for some p'' with $p'\mathcal{R}p''$, $p'' \Vdash x \in \hat{\omega}$, and hence for some h, $p'' \Vdash [\![x \approx h]\!]$ and $p'' \Vdash [\![h \,\varepsilon\, \hat{\omega}]\!]$ (Definition 1.6 Chapter 17). Then for some p''' with $p''\mathcal{R}p'''$, $p''' \Vdash h \,\varepsilon\, \hat{\omega}$, and so $\langle p''', h\rangle \in \hat{\omega} = \mathcal{G} \times \{\hat{0}, \hat{1}, \ldots\}$. Then $h = \hat{n}$ for some $n \in \omega$. It follows that $p'' \Vdash [\![x \approx \hat{n}]\!]$ and $p'' \Vdash [\![\hat{n} \in a]\!]$.
2. Now let $b = \{\langle q, \hat{n}\rangle \mid n \in \omega, q \Vdash [\![\hat{n} \in a]\!]\}$. Trivially $b \subseteq \mathcal{G} \times \{\hat{0}, \hat{1}, \ldots\} = \hat{\omega}$.
3. $p \Vdash [\![a \subseteq b]\!]$. The proof is by contradiction. Suppose not; then for some h and for some p' with $p\mathcal{R}p'$, $p' \Vdash [\![h \in a]\!]$ and $p' \Vdash [\![\neg(h \in b)]\!]$ ($\mathbf{P}_9$, Page 226). By item 1, for some p'' with $p'\mathcal{R}p''$, $p'' \Vdash [\![h \approx \hat{n}]\!]$ and $p'' \Vdash [\![\hat{n} \in a]\!]$ for some $n \in \omega$. Let q be an arbitrary member of $\mathcal{G}$ with $p''\mathcal{R}q$. Then $q \Vdash [\![\hat{n} \in a]\!]$, hence $\langle q, \hat{n}\rangle \in b$, and so $q \Vdash \hat{n} \,\varepsilon\, b$. Since q was arbitrary, $p'' \Vdash \Box(\hat{n} \,\varepsilon\, b)$, and so $p'' \Vdash \Box\Diamond(\hat{n} \,\varepsilon\, b)$, or $p'' \Vdash [\![\hat{n} \,\varepsilon\, b]\!]$. Then $p'' \Vdash [\![\hat{n} \in b]\!]$ (Corollary 2.4, Chapter 17). But we also have $p'' \Vdash [\![\neg(\hat{n} \in b)]\!]$, and this is our contradiction.
4. $p \Vdash [\![b \subseteq a]\!]$. Again the proof is by contradiction. If not, then for some h and for some p' with $p\mathcal{R}p'$, $p' \Vdash [\![h \in b]\!]$ and $p' \Vdash [\![\neg(h \in a)]\!]$. Then for some p'' with $p'\mathcal{R}p''$, $p'' \Vdash h \in b$ and hence for some k, $p'' \Vdash [\![h \approx k]\!]$ and $p'' \Vdash [\![k \,\varepsilon\, b]\!]$. Then for some p''' with $p''\mathcal{R}p'''$, $p''' \Vdash k \,\varepsilon\, b$. But then $\langle p''', k\rangle \in b$, and so $k = \hat{n}$ for some $n \in \omega$, and $p''' \Vdash [\![\hat{n} \in a]\!]$ (definition of b). We also have $p'' \Vdash [\![h \approx \hat{n}]\!]$, and it follows that $p''' \Vdash [\![\neg(\hat{n} \in a)]\!]$, a contradiction.

Additional changes resulting from the correction described above.

Page 264, second paragraph should begin: "Let $C = \{a \mid a \subseteq \mathcal{G} \times \{\hat{0}, \hat{1}, \ldots\}\}$."

Page 264, last paragraph of the Proof of Lemma 5.2, second sentence. This should begin: "Since $a \subseteq \mathcal{G} \times \{\hat{0}, \hat{1}, \ldots\} \ldots$".

Page 265, paragraph following Proposition 5.5. This should read: "We are finished investigating $\mathcal{P}(\mathcal{G} \times \{\hat{0}, \hat{1}, \ldots\}$ and its subset C_0.

Page 272 (Problem found by Grigori Mints). The proof of Proposition 20.4.1 begins by noting that $f \in g$ and $[\![f \in g]\!]$ are equivalent. This is not the case, see correction to Page 228. However, the atomic case is still straightforward. For $f, g \in \mathcal{D}_{\mathfrak{F}}^{G}$, $p \Vdash f \in g$ if and only if $p \Vdash_{\mathfrak{F}} f \in g$ for every p, by the definition of $\Vdash_{\mathfrak{F}}$. It follows that $p \Vdash \Box\Diamond(f \in g)$ if and only if $p \Vdash_{\mathfrak{F}} \Box\Diamond(f \in g)$ for every p.

The following errors and typos were reported by Jonathan Farley.

Page 23, line 17 "aet" should be "set"

Page 25, line 14 "A2" should be "A5"

Page 30, line 19 "qualifiy" should be "qualify"

Page 38, line 2 It is true as written, but "$y \subset x$" should be "$y \subseteq x$"

Page 40, line 11 "bounded" should be "non-empty bounded"

Page 44, line 15 Do we know c is a set? Response: standard mathematical practice treats this as a set, but technically it is not justified until the Axiom of Substitution is introduced, **Ax 8** on page 170.

Page 49, line 14 "$x'Rb'$" should be "$b'Rx'$"

Page 49, line 15 "$x \leq b$" should be "$b \leq x$"

Page 58, line -2 "M" should be "N"

Page 59, line -5 "M" should be "S"

Page 60, line -8 "N should be "$\cup N$"

Page 60, line -5 (Further corrected by Rolf Rolles) This should read "Every successor element of N is $F(a)$ for some $a \in \cup N$."

Page 60, Exercise 4.3 Delete this exercise. Here is a counter-example. Take any set x, and consider $S = \{x\}$. The axiom of choice is not needed to say S has a choice function, but $\cup S = x$, and this need not have a choice function

Page 62, line 9 The semicolon should be a comma

Page 62, Lemma 5.4 Add the assumption that $A \neq \emptyset$

Page 62, line -2 "5.3" should be "5.2"

Page 63, line 5 Exercise 4.3 has been deleted

Page 63, line 7 After "denumerable" add "or finite"

Page 64, line -9 Before "set of finite character" insert "non-empty"

Page 66 It would have helped to point out at the beginning of §7 that g is defined on all sets

Page 73, line 3 "10.2" should be "10.3"

Page 79, following Definition 1.1 Should begin, "In general, $F(x) \neq F''(x)$"

Page 79, Second paragraph following Definition 1.1 Should contain "whereas $F''(x)$ is"

Page 80, line 2 Add that φ_2 is $1-1$

Page 80, proof of Proposition 1.3, second line "L" should be "$L_<$"

Page 80, line -2 "onto" should be "into"

Page 88, line -7 "isomporphic" should be "isomorphic"

Page 89, Theorem 6.1 This should begin "For any functions $f(x), g(x)$ on ordinals, and any function $h(x, y, z)$, where y and z are ordinals, ...

Page 93, $\mathbf{O}_6$ "Since x" should be "Since S"

Page 97, last line of Proof of $\mathbf{Q}_4$ "has rank $< \alpha$" should be "has rank $\leq \alpha$"

Page 97, line -8 "*mathbbF*" should be "$\mathbb{F}$"

Page 99, next to last line of Example "every subclass" should be "every non-empty subclass"

Page 99, line -14 "Zermelo Fraenkel" needs a hyphen

Page 101, line -16 "x of A" should be "x of $A - B$"

Page 102, line -6 "set" should be "class"

Page 306, Tarski, A. (1955) "lattice-theoretical theorem" should be "lattice-theoretical fixpoint theorem"

The following corrections are due to Fausto Barbero.

Page 104 In the proof of Theorem 4.6, it should be $y = g''x$, not $y = g(x)$.

Page 116 This is not actually a correction, but an elucidation. The second paragraph on the page, concluding $A \cong A \times A$ from $M \cong A$ and $M \cong M \times M$, does not look obvious at all to me. I give a short proof.

First, we observe that $A \times A \cong [(A \setminus M) \cup M] \times [(A \setminus M) \cup M] = [(A \setminus M) \times (A \setminus M)] \cup [(A \setminus M) \times M] \cup [M \times (A \setminus M)] \cup [M \times M]$. But $M \cong A$ implies $A \setminus M \preceq M$. Using this injection, it is immediate to prove that each of $[(A \setminus M) \times (A \setminus M)], [(A \setminus M) \times M]$ and $[M \times (A \setminus M)]$ is $\preceq M \times M$. Then, using the law of additive absorption (Thm 6.2) in the first equation, $A \times A \cong M \times M \cong M \cong A$.

Page 123 In the first line of the proof of Proposition 9.8, instead of c there should be x.

Page 124 Line -3: instead of $z = y \cup \omega$ it should be $z = \mathcal{P}(y \cup \omega)$.

Page 132 Subsection "A-ranks", third line: instead of $p(x) \cap A$ it should be $\mathcal{P}(x) \cap A$.

Page 133 Last line on page: instead of $F_\alpha(y)$ it should be $f_\alpha(y)$.

Page 139 Proof of 6.7, first line: "Let $A \ldots$" should be "Let $A_0 \ldots$".

Page 144 Paragraph beginning "One of the early major results...," the reading should be "... if φ is satisfiable in any INFINITE relational system at all, then it is satisfiable in a denumerable relational system" (remember that "denumerable" means having cardinality ω).

Page 153

- Line 4 of (4): instead of w_n, it should be W.
- Lines 17, 19 and -7 of (4): terms of the form $\cup\Gamma''(w_i)$ are used, but they are not well defined, since Γ is an n-ary function. Could one use $\cup\Gamma''(w_i, \ldots, w_i)$ instead?

- Not an error, but it is worth noting that n is used for two distinct purposes in this proof.

Page 163 The formula (29) seems incorrect, because it should involve two (in general) distinct pairs $\langle x_1, x_2\rangle$, $\langle x_1, x_3\rangle$, but this is not possible if x_1, x_2, x_3 are all extracted from members of one and the same z (remember that z should be one of the pairs that constitute the function x). The correct prefix of the formula should be:

$$(\forall z_1 \in x)(\forall z_2 \in x)(\forall w_1 \in z_1)(\forall w_2 \in z_2)(\forall x_1 \in w_1)(\forall x_2 \in w_1)(\forall x_3 \in w_2)$$

Page 167 I have the impression that the formula used to define $L^n \cap c$ is based on the (generally wrong) assumption that $L^n \cap c$ is transitive. It should be replaced by $x \in c \wedge (\exists x_1 \in_2 c)(\exists x_2 \in_4 c) \ldots (\exists x_n \in_{2n} c)(x = \langle x_1, \ldots, x_n\rangle)$, where $y \in_k z$ abbreviates

$$\exists y_1 \ldots \exists y_{k-1}(y \in y_1 \wedge y_1 \in y_2 \wedge \cdots \wedge y_{k-1} \in z).$$

Page 170 It is not specified anywhere that the axioms are the universal closures of the indicated formulas. Furthermore, in Axiom 2, $\varphi(x, y_1, \ldots, y_n)$ should be replaced (twice) by $\varphi(y, y_1, \ldots, y_n)$.

Page 176 Proof of 3.4: replace "non-empty member of L" with "non-empty member a of L".

Page 183 A further lemma needed to show that formulas stay Σ if the symbol $\upharpoonright$ is used.

Page 186 In clause (0) there are two occurrences of substitution of a set inside a term; but this has not (yet) been defined.

Page 187 In the definition of $x(y) = z$ just before Proposition 3.9, "$\varphi(y) = z$" should be replaced by "$\varphi(y) =$ the formula whose code is z". In "Valuations and truth", the index j is introduced but never used.

Page 189 Line 13, twice: replace "absolute" with "absolute over L". However, the same argument (and thus Theorem 4.1) should work for any first-order universe. Indeed, on **Page 190**, line -9, Theorem 4.1 is applied to a generic first-order universe K.

Page 196 I think condition C_1 should be $K_0 = \emptyset$. Otherwise it would be impossible that $\pi''(K_0) = L_0$, as claimed in Exercise 2.1. In condition C_3, $\alpha < \delta$ should appear as an index.

Page 210 Line -3: The exclamation mark is unintentional.

Page 217 Before Prop. 4.4: The last occurrence of "than" should be "them".

Page 219 Line 3: The argument also uses the Converse Barcan Formula.

Page 242 Most likely, the proof of 6.1 should begin "$u = \{q, a\}$" and "$v = \{q', b\}$".

Page 256 Proof of Lemma 6.1, penultimate paragraph: "Then both...": this is not really an error, but the existence of a common p' satisfying both the conditions described is not straightforward as the text seems to imply. I think one should proceed as follows: from the fact that $p \Vdash [\![(\exists y)(y \text{ is } f(\hat{\gamma}_1) \wedge y \in s)]\!]$ and $p \Vdash [\![(\exists y)(y \text{ is } f(\hat{\gamma}_2) \wedge y \in s)]\!]$ we get (by Prop. 4.3 (3)) that $p \Vdash [\![(\exists y)(y \text{ is } f(\hat{\gamma}_1) \wedge y \in s) \wedge (\exists y)(y \text{ is } f(\hat{\gamma}_2) \wedge y \in s)]\!]$. Then, applying prenex transformations, we obtain that $p \Vdash [\![(\exists y)(\exists y')((y \text{ is } f(\hat{\gamma}_1) \wedge y \in s) \wedge (y' \text{ is } f(\hat{\gamma}_2) \wedge y' \in s))]\!]$. Applying twice the embedding property $\mathbf{P}_{11}$, we obtain p', a, b such that $p' \Vdash [\![(a \text{ is } f(\hat{\gamma}_1) \wedge a \in s) \wedge (b \text{ is } f(\hat{\gamma}_2) \wedge b \in s)]\!]$. Using once more Prop. 4.3 (3) we obtain the desired conclusion.

Page 257 In order to account for the special case CH, in both centered formulas of the proof of 6.2 "$\hat{\omega} \in \alpha$" should be replaced by "$\hat{\omega} \in \alpha \vee \hat{\omega} \approx \alpha$".

Page 260 Two lines before the "Exercises": instead of ω it should be $\hat{\omega}$.

Page 299 Line -12: "computibility" should be "computability".