

## A Tribute to Professor Helena Rasiowa Melvin Fitting

We who work in science like to think the world is one—the world of science itself, which is universal and has no boundaries. Of course, it is not so. The real world is divided politically, economically, and by the artifacts of history. Professor Rasiowa and I came from two quite different worlds. My world being what it was, it took me a long time to understand this.

My world was one of opportunity and privilege, personally unearned. I was a student in the cold war United States, a country spurred by Sputnik to help its scholars. At the time, I didn't know how fortunate I was. Professor Rasiowa's world, by contrast, was one that required struggle and courage. Eventually I learned something of this, even as conditions were easing.

My first encounter with the work of Professor Rasiowa was through the famous book, "The Mathematics of Metamathematics," written with Roman Sikorski and published in 1963. I no longer know what prompted me to read the book while in graduate school, but the effect of doing so was powerful. Indeed, I quickly bought my own copy (second edition, 1968), which I still happily have. It was a strange book. On the one hand, it was painfully exhaustive and thorough in the presentation of material. Page after page enumerated detailed results almost in the form of a catalog, dry and utilitarian. On the other hand, the ideas were of a sort I had never come across before, and I was enchanted. This way of using algebra, producing known results in classical logic, then applying similar techniques to non-classical logics to get new results—it all seemed like magic. Profound results fell out so effortlessly, it seemed. How could one read this book and remain unaffected?

The algebraic approach to logic, properly speaking, goes back to George Boole, but its modern incarnation claims Tarski as founder. Tarski's ideas, in turn, were furthered by Mostowski, who had both Rasiowa and Sikorski as students. It is not surprising that, among logicians of my generation, Poland and algebra were thought of as synonymous.

The central theorem of the book is entirely algebraic in nature, and comes from an earlier Rasiowa, Sikorski paper of 1950. To explain its role, one must know a little about the algebraic approach to completeness proofs.

For an axiomatically formulated propositional classical logic it is not hard to show the Lindenbaum algebra is a Boolean algebra, with the set of theorems as the unit. Consequently if a formula is not a theorem, it belongs to some member of the Lindenbaum algebra that is not the unit. Now for the

completeness theorem the key is to show that, given any Boolean algebra, and any member  $a$  other than the unit, there is a homomorphism to the two-element boolean algebra  $\{\perp, \top\}$  that maps  $a$  to  $\perp$ . This, in effect, produces a two-valued truth assignment that corresponds to a conventional truth table. Combining all this, any non-theorem can be falsified. Showing the homomorphism result is equivalent to showing that in any Boolean algebra, for any non-unit member  $a$  there is a maximal ideal  $\Delta$  containing  $a$ , and this in turn is equivalent to showing that for any non-zero member  $a$  there is a maximal filter  $\nabla$  containing  $a$ .

For first-order classical logic, the Lindenbaum algebra also exists, and it plays the same role that it did propositionally. But quantifiers correspond to infinitary meets and joins, so something stronger than a Boolean algebra must be considered. The difficulty is, a homomorphism theorem, or a maximal ideal theorem, or a maximal filter theorem taking infinitary operations into account cannot be proved. What Rasiowa and Sikorski showed was that, if one is only interested in a given *countable* collection of meets and joins, a generalization of the maximal filter theorem holds. This is enough to establish the Gödel completeness theorem algebraically.

The Rasiowa-Sikorski theorem was proved by them using topological methods—specifically, the Baire category theorem. Indeed, they are essentially equivalent. With a date of 1950, it was emphatically part of a worldwide shift in the treatment of logic: it was now part of mathematics, and could be treated just like any other part. Rasiowa and Sikorski were quite conscious of this—note the title of their book. And they made it explicit in the Preface, “We have given very little space to set-theoretical and semantic antinomies. We believe that antinomies should be relegated to the history of mathematics. . .”. The Rasiowa-Sikorski theorem is a fundamental result. Further, it is one of a handful of results in which countability plays a fundamental role. In logic, countability also appears in the omitting types theorem, and in the existence of generic sets in forcing. The Rasiowa-Sikorski theorem is not as well-known as these others, but deserves to be.

I once heard Dana Scott criticize “The Mathematics of Metamathematics” because, while it took an algebraic approach to logic, it did not carry the work further and consider set theory. If it had, forcing would have been discovered years earlier than it was. This is not, at heart, a criticism, but a tribute. The building of mathematics always goes on. Foundations, firmly laid, enable later construction, and the foundations laid by that book were powerfully firm.

Reading “The Mathematics of Metamathematics” as a graduate student,

I was entranced. I knew nothing of the authors. Coming from the world I did, I didn't begin to suspect the difficulties that faced the authors were much more than mathematical. The theorem was published in 1950. In 1944 Poland saw the Warsaw Uprising, during the Nazi occupation. As a consequence, Warsaw was systematically destroyed. Rasiowa's Master's thesis was burned, along with the house, and she and her mother survived in a cellar beneath the ruins. Indeed the writing of a Master's thesis itself was peril because, under Nazi occupation, the very process of higher education became conspiracy. How long a distance, and how short a time, from there to the end of the war, and the defense of her PhD thesis in 1950!

Above I described the algebraic approach to classical logic. Rasiowa continued this work in her later book, "An Algebraic Approach to Non-Classical Logics," published in 1974. But the Rasiowa-Sikorski book itself also presented algebraic approaches to intuitionistic and some modal logics. My PhD dissertation made use of this work on intuitionistic logic, and extended it somewhat. It was published in book form in 1969. I thought of my work as being about set theory, but everybody else apparently thought of it as being about intuitionistic logic. At any rate, it seems to have called me to the attention of some of the logicians in Warsaw, because within a few years I got an invitation to visit. I said no. I was young, with a new family started, with little time or money. But after the invitation was repeated, I agreed, and went to Warsaw for week during a Banach semester, where I first met Professor Rasiowa.

By this time the hot war was long over, and the cold war firmly in place. While not impossible, it was difficult for Poles to get around the world, but it was certainly possible to bring the world to Poland. That seems to have been a purpose of the Banach semester. Professor Rasiowa had taken on necessary roles of administration and ambassadorship. As I gradually came to understand, it took a powerful force to keep Polish mathematical life independent, and make it serve as a bridge between the Russian mathematical community and that of the west. Russian work was often invisible in the west. Perhaps it was true the other way around as well; I don't know. Poland faced both ways and thus could play a unique and important role. Throughout her career Professor Rasiowa served in various ways within Poland (Dean, positions in the Polish Mathematical Society, etc.), and also in roles connecting Poland with the west (in the Association for Symbolic Logic, in the International Union of History and Philosophy of Sciences, etc.).

I do not know details of the history of this period in Poland. But I understand enough to recognize the importance of Polish journals, partic-

ularly *Studia Logica* and *Fundamenta Informaticae*. Among other things, these served to keep the Polish logic community in a strong position with respect to the rest of the world. Professor Rasiowa served as Collecting Editor for the first of these for many years, helped found the second, and served as its Editor-in-Chief. She also traveled extensively, thus maintaining a world-wide web of contacts.

Earlier than many, Professor Rasiowa saw the coming importance of computer science as a force in mathematics. It was a source of problems that were akin to those logic traditionally dealt with. Algebraic methods applied naturally and, during the later part of her career, this area was central in her research. Looking at her bibliography, I am struck by the number of papers touching on computer science, and the list of her collaborators is especially noteworthy. It shows the international role that Professor Rasiowa played. I see Nguyen Cat Ho of Vietnam, Wiktor Marek, originally of Poland, now of the United States, George Epstein, of the United States, Andre Skowron of Poland, and others. And the areas: algorithmic logic (formal logic of programs), rough sets (approximating to concepts), intelligent agents and consensus, fuzzy logics.

Over the years I met Professor Rasiowa several more times, and corresponded with her. I returned to Poland for her retirement. She visited the United States many times, and I gave a small party for her once at my house. Partly through her efforts I became involved with both *Studia Logica* and *Fundamenta Informaticae*. She passed on to me a graduate student, for which I will always be grateful. While I don't know for certain, I suspect she did similar things with many people during her extensive travels. She always struck me as a quiet, dignified lady, polite and pleasant. But as I came to understand eventually, besides her mathematical ability, she had the necessary strength and skill to be a central figure, helping to hold together and build a mathematical community under difficult circumstances. It is a rare talent and, sadly, it was a needed one. Not just Poland, but the world logic community, was the beneficiary.