A Mistake on My Part

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1 The Background

I first met Dov at a logic conference in Manchester, in August 1969, though we had begun a mathematical correspondence the previous year. Here are a few photos from the conference. As I recall, Dov planned to get married shortly after the conference. I found a letter in my files mentioning that I sent Dov a copy of the pictures in 1969, so this is for everybody else.

Figure 1 shows Dov at Jodrell Bank, the huge radio telescope complex run by the University of Manchester.

Figure 1.

Figure 2 shows, from left to right, Saul Kripke, Dov, Michael Rabin, and someone I can't identify.

Figure 2.

Figure 3 shows the following. Back row: David Pincus, Miriam Lucian, Dov, Saul Kripke; front row: George Rousseau, and me.

It was also at this conference that a mistake I made got straightened out. I don't know if it had an effect on Dov's work, but it may have. But let's discuss that in a section of its own.

2 Where I Went Wrong

Kripke's semantics for intuitionistic logic came along in [Kripke, 1965], and prompted a considerable amount of research. It was a possible worlds semantics, and in it domains for quantifiers varied from world to world, though subject to a monotonicity condition. It was a natural question: what would a restriction to constant domain intuitionistic models impose. In my dissertation, [Fitting, 1969], I had shown in passing that, for formulas without universal quantifiers, constant domain and monotonic domain semantics validated the same formulas (unlike in classical logic, the two quantifiers are not interdefinable intuitionistically). This is not the case once universal quantifiers are present, so just what is the logic of constant domain Kripke intuitionistic models. For modal logics, Kripke had addressed an analogous problem by showing constant domains correspond to the Barcan formula, but the question was still open for intuitionistic logic.

Dov asked me, in a letter, about a conjectured axiomatization for the

Figure 3.

constant domain version of intuitionistic logic. I replied on March 13, 1969, and my response was quite decisive. Here is an excerpt from my letter to Dov.

> ...However, I can settle one of the problems you raised in your letter.
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> You asked does the following axiomatize constant domains:
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> (1) $(\forall x)[P(x) \vee Q(x)] \supset [(\exists x)P(x) \vee (\forall x)Q(x)]$.
>
> No. About a month ago I began working with intuitionistic logic plus the axiom
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> (2) $(\forall x)[A \vee B(x)] \supset [A \vee (\forall x)B(x)]$,
>
> where x does not occur free in A, and it has turned out to be a most interesting system. Schemas (1) and (2) are equivalent. If, in (1) we let $P(x)$ be A, and $Q(x)$ be $B(x)$, (1) is (2). If, in (2) we let A be $(\exists y)P(y)$, and $B(x)$ be $Q(x)$, (2) implies (1).
>
> Call intuitionistic logic **I**, and call **I** plus (2) (or (1)), **SI**. A simple way to characterize **SI** is by Beth tableaus. Use the system of *The Foundations of Mathematics* section 145, but replace rule vi^b by the corresponding classical rule, vi^b of section 92 (then all quantifier rules are classical). This is **SI**.
>
> **I** can be embedded in **SI** as follows. Let *true* be a truth constant (e.g. let *true* be $X \supset X$). For any formula F, define a translate,

F^* to be like F except that subformulas of the form $(\forall x)A(x)$ are replaced by $\mathit{true} \supset (\forall x)A(x)$. Then

(3) $\vdash_{\mathbf{I}} F$ if and only if $\vdash_{\mathbf{SI}} F^*$.

To return to your original question, the translate of (2) is not provable in **SI**, but is valid in all constant domain models.

The Beth tableau system hybrid referred to above is perhaps more conveniently seen as the combination of an intuitionistic propositional system using signed formulas, described in [Fitting, 1983; Fitting, 1998], with the classical rules of Smullyan, from [Smullyan, 1968].

3 What Was Wrong

I wonder if my negative March response stopped Dov from following up on the conjectured axiomatization. At any rate, he did not do so. But Sabine Görneman was present at the August Manchester conference, and she had just proved that (2) was precisely the axiom one needed to add to intuitionistic logic to axiomatize constant domain models. Her dissertation containing this result was written later—I received my copy in February of 1970, and a paper based on it appears as [Görneman, 1971]. I well remember Dov devoting much of a conference bus trip, to a nearby site of interest, to walking up and down the aisle and trying to reconstruct Görneman's proof. Apparently he succeeded, because in her Journal of Symbolic Logic paper, Görnemann mentions "another proof has been given by D. Gabbay." The citation is [Gabbay, 1969], but I have never seen this.

Equivalence (3) is correct, and so the translate of (2) is, indeed, not provable in the tableau system, since (2) is not an intuitionistic theorem. What is not true is my assertion that intuitionistic propositional plus classical quantifier tableau rules give a proof procedure equivalent to axiomatic intuitionistic first-order logic plus (2). In fact, all the axioms are provable in the tableau system. What we have here is a seemingly natural set of tableau rules for which cut elimination does not hold. Indeed, this should have been seen even without Görneman's crusher. Formula (2) is provable in the tableau system, but its translate is not. However, a formula and its translate differ by having some subformulas Z replaced with subformulas $\mathit{true} \supset Z$, and the equivalence of these can be proved using just intuitionistic propositional tableau rules. If cut elimination held, one could prove that (2) and its translate were provably equivalent, so both or neither should have been provable.

Well, I was young and careless. I'm older now. But even if Dov did not get this result first, there have certainly been many other firsts over a long career. Happy birthday, Dov.

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