Editorial

The title of this Journal is *Logic and Computation*. Properly seen, a computation is a kind of formal proof, and so is subsumed under the heading of logic. Certainly the connective 'and' is part of the subject matter of logic as well, so the title of the Journal may be reduced simply to *Logic*. It is Logic, singular and plural, that I wish to discuss.

The term 'logic' has varied in its meaning over the years. For Isaac Watts (as quoted by George Boole), "Logic is the art of directing the reason aright in acquiring the knowledge of things, for the instruction both of ourselves and the others", and thus it is a kind of road map. At another extreme, for chip designers logic is essentially a circuit diagram (once again a kind of road map).

Let us confine our discussion to symbolic logic in the twentieth century. Even with the scope thus narrowed, we still see non-uniform usage. The term 'symbolic logic' (or just 'logic') has become more encompassing throughout the century, roughly paralleling in a compressed time span a similar broadening in the technical usage of the work 'algebra'. At the start of the century Bertrand Russell could write simply, "The subject of Symbolic Logic consists of three parts, the calculus of propositions, the calculus of classes, and the calculus of relations".[1] Near the end of the century we find several books on the market with the plural 'logics' in their title. What happened?

For Russell logic was a single entity with three parts, like Gaul. To be sure, it was broad enough to encompass all mathematics. (Or rather, it was thought to be, but was not. That's another story.) It would have puzzled Russell to hear talk of logics. It would be akin to the classic test question, "Define the universe and give three examples". After all, "Die Welt ist alles, was der Fall ist". Russell recognized that logic was not monolithic, but the divisions he recognized formed a strict hierarchy, ranging from propositional logic upwards. There were no side roads. This is comparable to the early conception of algebra as the study of the properties of numbers. Since the number systems form a hierarchy, this produces a simple hierarchy of algebras. In the early nineteenth century there were no alternative algebras.

Boole seemed a little more generous in conception than Russell. "In saying that it is conceivable that the law of thought might have been different from what it is, I mean only that we can frame such an hypothesis, and study its consequences. The possibility of doing this involves no such doctrine as that the actual law of human reason is the product either of

[1] *The Principles of Mathematics*, Bertrand Russell, first edition 1903, second edition 1938, page 11.

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chance or of arbitrary will".[2] It may be no coincidence that Boole was one of those (like Hamilton with quaternions) who forced a broadening of the notion of algebra.

Within a few years of the publication of *Principia Mathematica,* C. I. Lewis was already considering alternative logics involving additional connectives, $\Box$, $\Diamond$, and even a kind of compatibility connective, $\bigcirc$. These remained merely a curiosity among the general public for a long time. But there was also Brouwer. If logic was "the fundamental laws of those operations of the mind by which reasoning is performed", as Boole said,[3] Brouwer argued that the intended subject matter was not properly captured in the system of *Principia Mathematica* and its kin. His student Heyting formulated a symbolic logic, today known as Intuitionistic Logic, or sometimes the Heyting Predicate Calculus. From here on, even without the additional operators of Lewis, there were alternative logics vying for attention. Whatever the philosophical claims of Russell and Brouwér, logic had inescapably become plural.

Once the conceptual dam had burst, alternative alternatives were not long in coming. Over the years the following logics have been introduced (this is not meant to be an exhaustive list, merely an exhausting one).

- Minimal logic
- Negationless logic
- Infinitary logics
- Free logic
- Quantum logic
- Modal logics
- Temporal logics
- Dynamic logic
- μ-calculus
- Process logic
- Many-valued logics
- Relevance logics
- Linear logic
- Non-monotonic logics

By the end of the twentieth century, formal logic has become a big family. Notice how many of the items on the list above either were created with computer science applications in mind, or developed such applications afterwards.

While there is something of a family resemblance, still may be the best characterization of logic is: it's what logicians do. (For a comparable problem, try characterizing algebra without using the negative approach of saying it's not analysis.) Well, what is it that logicians do? What is peculiar about us? (an invitation to a rude answer if I ever heard one). A logician, like an algebraist, has some class of objects in mind for investigation, say groups. Both specify their subject matter in a similar way—by giving axioms for groups. But for the algebraist, the group axioms are incidental; they

[2] *An Investigation of the Laws of Thought,* George Boole, 1854, footnote page 50.
[3] *ibid,* page 1.

serve merely to specify the topic. For the logician, the axioms themselves are part of the subject, and their syntactic features may be reflected in features significant to the algebraist. For example, everybody knows that the homomorphic image of a group is a group. One can find a direct proof of this in any elementary algebra book. But in a logic book one might find the more general result: the homomorphic image of a model for a set S of positive first-order formulas is another model for S. Then one merely observes that the usual group axioms are positive formulas of first-order logic with equality. What the logician has added is a connection between an algebraic property of groups, and syntactic features of the very definition of group. It is this that distinguishes the logician. If you describe something, you do so in a language, and details of the language are important.

Today we have moved a long way from Russell at the beginning of the century. Now, especially in computer science, people design new logics freely. Is this wise? The blunt and forthright answer is: it depends. The logics listed above all have certain common features. Consider them.

Each logic is, in an informal sense, about some recognized phenomenon, human, mechanical, physical. In each case the intended subject matter is restricted, coherent, and of interest to many people. Briefly, the logics are simple and natural. (I know these are ill-defined notions.)

Each logic has a good syntax. The machinery of language is never more complicated than it has to be to treat the subject. As far as we are concerned here, logic is a human tool and its language must be easily comprehended. To use a programming metaphor, something between APL and COBOL is nice.

Each logic has a useful semantics, and this semantics reflects the intended subject. Actually, there are exceptions. There is considerable discussion about whether linear logic, for instance, has a semantics that meets this condition—still, it does have semantics that are useful mathematically. Modal logics, for many years, did not have a semantics. But the exceptions, and the work that went into removing them as exceptions, shows the importance attached to having a good semantics.

Each logic has a proof procedure. These proof procedures not only allow us to establish truths of the logic, but also serve as a tool to reason about it. (Think of the cut-free sequent calculus for classical logic.) Again there are exceptions. The μ-calculus lacks a proof procedure, though there are candidates. Infinitary logics have proof procedures that are not finitary. Once again, the prominence of the exceptions serves to show how important a good proof procedure is rated.

Perhaps someday there will be a subject, universal logic, as there is already universal algebra. There has been an attempt at it,[4] but while

[4] Axioms for abstract model theory, K. J. Barwise, *Annals of Mathematical Logic*, vol. 7, pp. 221–265, 1974.

interesting, the range was small. In the meantime, I expect computer science to be the incubator of new logics. In the past this role was played by philosophy or pure mathematics, or even physics; for the forseeable future, the job is probably ours. One should not feel constrained to fit things into the framework of classical logic, or even into the framework of logics already existing. A logic can be tailored to fit a subject as well as the other way around. But remember, in developing a new logic you are proposing a new candidate for a club whose members are noted for elegance and beauty. Remember the informal list of requirements for a logic that I discussed above. And let me add one more item to the list; again I quote Isaac Watts.[5] "You know, Sir, the great Design of this noble Science is to rescue our reasoning Powers from their unhappy Slavery and Darkness; and thus with all due Submission and Deference it offers a humble Assistance to divine Revelation". Meet this condition too, and you've got yourself a logic.

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[5] *Logic: or The Right of Reason,* Isaac Watts, 1726.